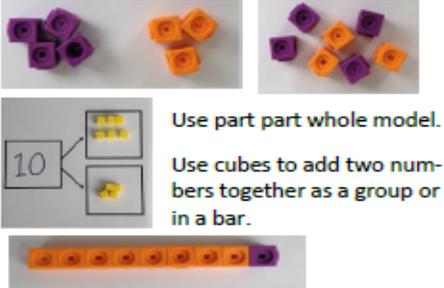
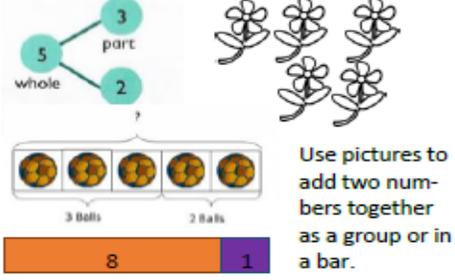
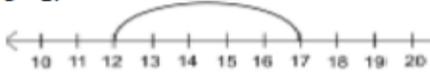
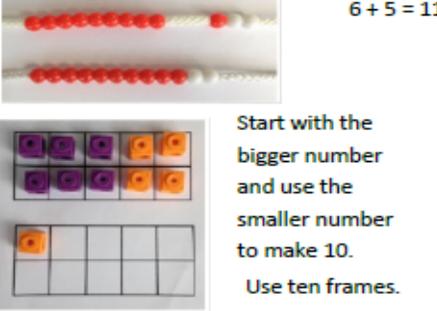
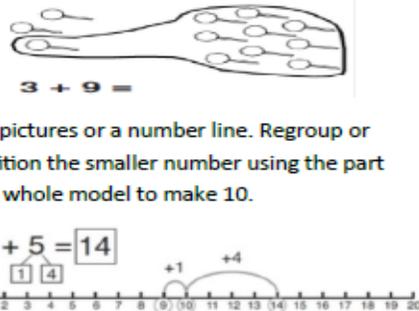
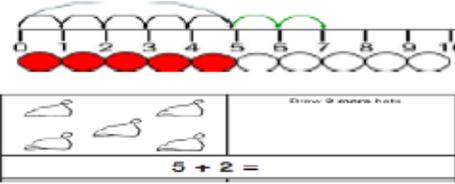
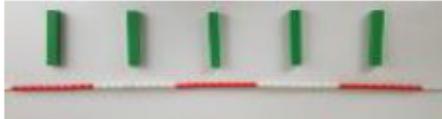
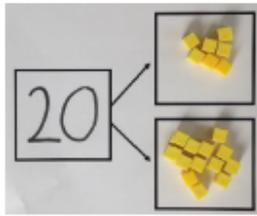
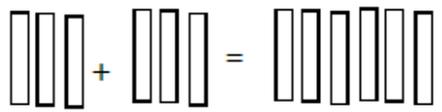
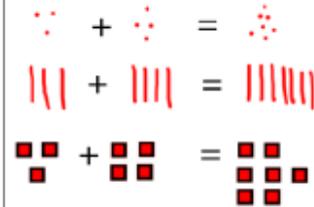
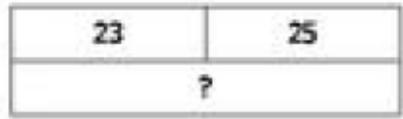




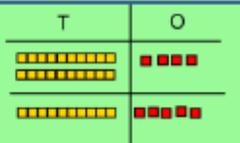
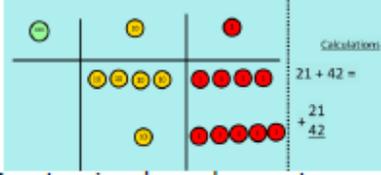
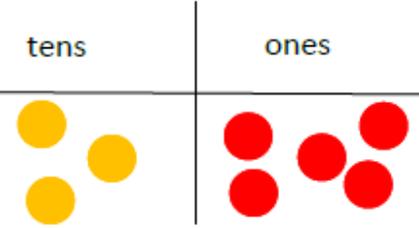
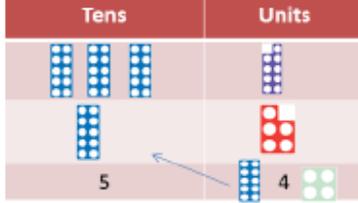
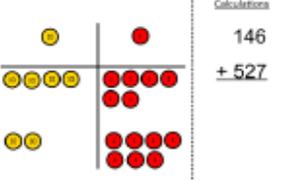
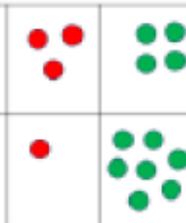
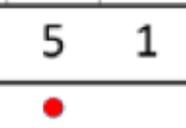
Objective & Strategy	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part- whole model	 <p>Use part part whole model.</p> <p>Use cubes to add two numbers together as a group or in a bar.</p>	 <p>Use pictures to add two numbers together as a group or in a bar.</p>	$4 + 3 = 7$  <p>Use the part-part whole diagram as shown above to move into the abstract.</p> $10 = 6 + 4$
Starting at the bigger number and counting on	 <p>Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.</p>	$12 + 5 = 17$  <p>Start at the larger number on the number line and count on in ones or in one jump to find the answer.</p>	$5 + 12 = 17$ <p>Place the larger number in your head and count on the smaller number to find your answer.</p>
Regrouping to make 10. <i>This is an essential skill for column addition later.</i>	 <p>Start with the bigger number and use the smaller number to make 10.</p> <p>Use ten frames.</p>	$3 + 9 =$  <p>Use pictures or a number line. Regroup or partition the smaller number using the part part whole model to make 10.</p>	$7 + 4 = 11$ <p>If I am at seven, how many more do I need to make 10. How many more do I add on now?</p>
Represent & use number bonds and related subtraction facts within 20	 <p>2 more than 5.</p>	 <p>5 + 2 =</p>	Emphasis should be on the language '1 more than 5 is equal to 6.' '2 more than 5 is 7.' '8 is 3 more than 5.'

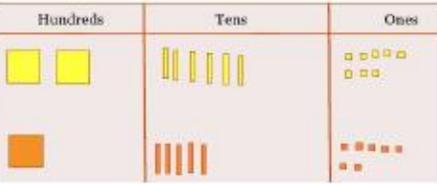
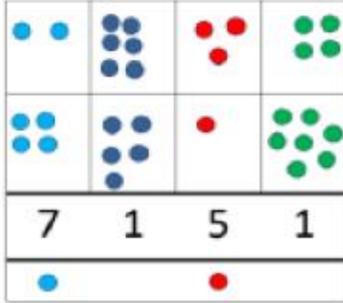
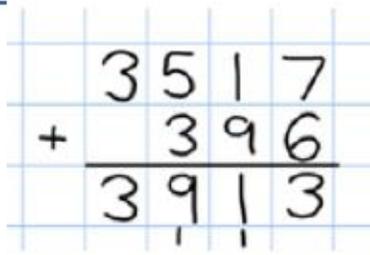
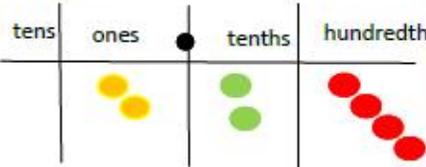
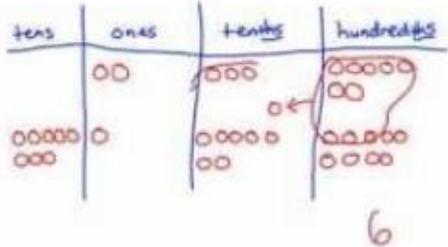
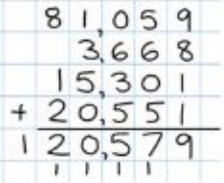
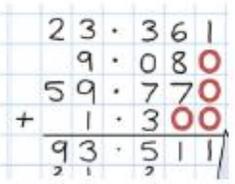
Y1 ADDITION +

Y2 ADDITION +

Objective & Strategy	Concrete	Pictorial	Abstract
Adding multiples of ten	$50 = 30 + 20$  Model using dienes and bead strings	 $3 \text{ tens} + 5 \text{ tens} = \underline{\quad} \text{ tens}$ $30 + 50 = \underline{\quad}$ Use representations for base ten.	$20 + 30 = 50$ $70 = 50 + 20$ $40 + \square = 60$
Use known number facts <i>Part part whole</i>	 Children explore ways of making numbers within 20	 $\square + \square = 20$ $20 - \square = \square$ $\square + \square = 20$ $20 - \square = \square$	$\square + 1 = 16$ $16 - 1 = \square$ $1 + \square = 16$ $16 - \square = 1$
Using known facts	$\square\square + \square\square = \square\square\square\square$ 	 Children draw representations of H,T and O	$3 + 4 = 7$ <i>leads to</i> $30 + 40 = 70$ <i>leads to</i> $300 + 400 = 700$
Bar model	 $3 + 4 = 7$	 $7 + 3 = 10$	 $23 + 25 = 48$

Y3 ADDITION +

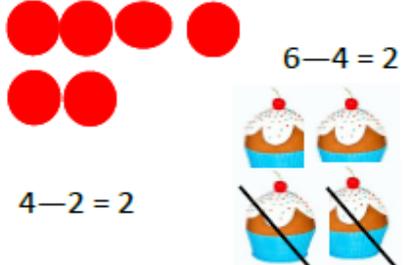
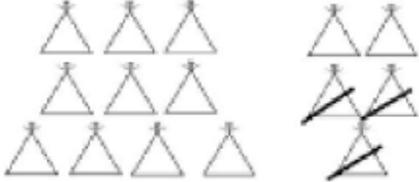
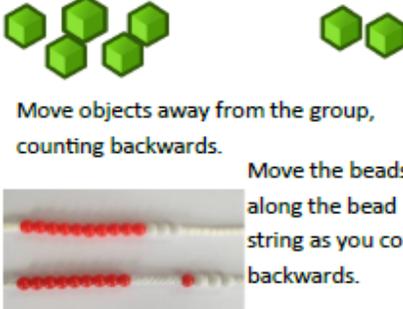
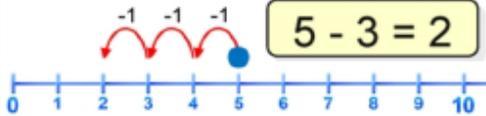
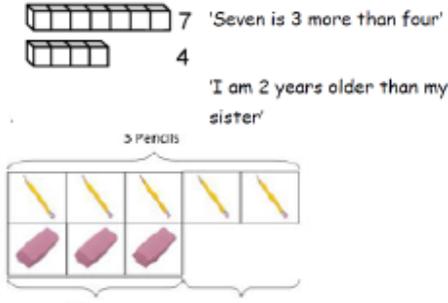
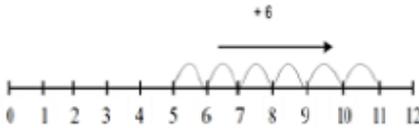
Objective & Strategy	Concrete	Pictorial	Abstract
<p>Column Addition—no regrouping (friendly numbers)</p> <p>Add two or three 2 or 3-digit numbers.</p>	<p>Model using Dienes or Numicon</p>  <p>Add together the ones first, then the tens.</p>  <p>7 9</p>  <p>Move to using place value counters</p>	<p>Children move to drawing the counters using a tens and one frame.</p> 	$\begin{array}{r} 223 \\ + 114 \\ \hline 337 \end{array}$ <p>Add the ones first, then the tens, then the hundreds.</p>
<p>Column Addition with regrouping.</p>	 <p>5 4</p> <p>Exchange ten ones for a ten. Model using Numicon and pv counters.</p>  <p>Calculations</p> $\begin{array}{r} 146 \\ + 527 \\ \hline \end{array}$	 <p>Children can draw a representation of the grid to further support their understanding, carrying the ten <u>underneath</u> the line</p> 	$\begin{array}{r} 20 + 5 \\ 40 + 8 \\ 60 + 13 = 73 \end{array}$ <p>Start by partitioning the numbers before formal column to show the exchange.</p> $\begin{array}{r} 536 \\ + 85 \\ \hline 621 \\ 11 \end{array}$

Objective & Strategy	Concrete	Pictorial	Abstract
<p>Y4—add numbers with up to 4 digits</p>	<p>Children continue to use dienes or pv counters to add, exchanging ten ones for a ten and ten tens for a hundred and ten hundreds for a thousand.</p> 	 <p>Draw representations using pv grid.</p>	 <p>Continue from previous work to carry hundreds as well as tens.</p> <p>Relate to money and measures.</p>
<p>Y5—add numbers with more than 4 digits.</p> <p>Add decimals with 2 decimal places, including money.</p>	<p>As year 4</p>  <p>Introduce decimal place value counters and model exchange for addition.</p>	<p>2.37 + 81.79</p> 	$\begin{array}{r} 72.8 \\ + 54.6 \\ \hline 127.4 \\ 11 \end{array}$ 
<p>Y6—add several numbers of increasing complexity</p> <p>Including adding money, measure and decimals with different numbers of decimal points.</p>	<p>As Y5</p>	<p>As Y5</p>	 <p>Insert zeros for place holders.</p> 

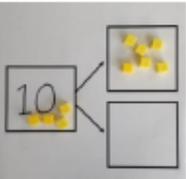
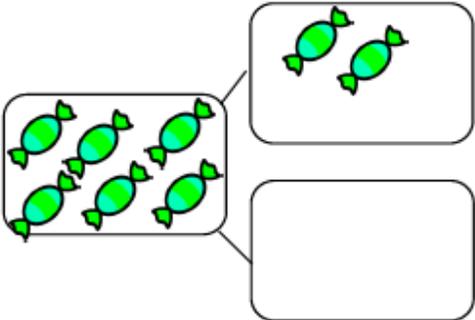
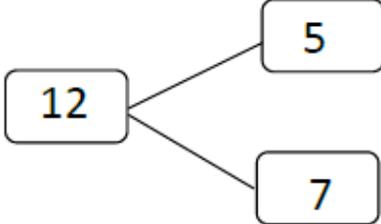
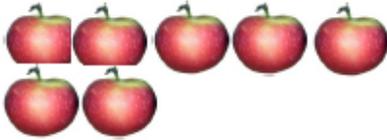
Y4-6

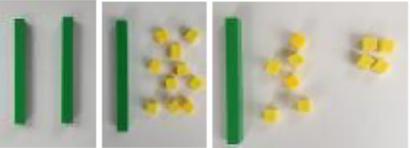
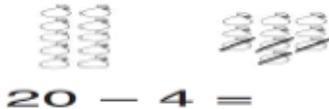
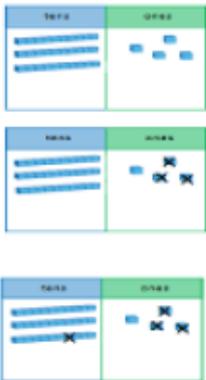
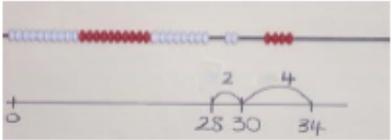
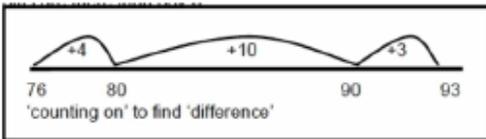
ADDITION +

Y1 SUBTRACTION -

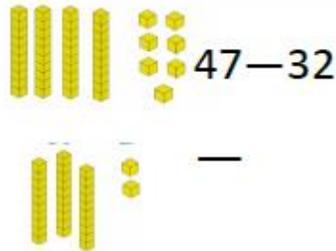
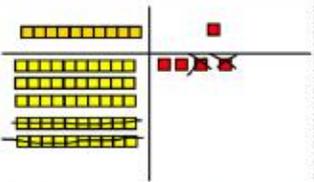
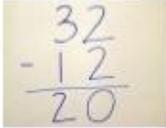
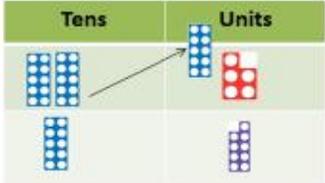
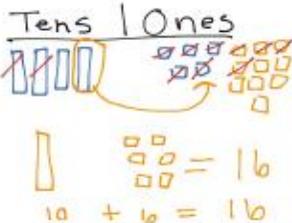
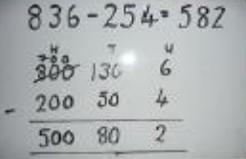
Objective & Strategy	Concrete	Pictorial	Abstract
Taking away ones.	<p>Use physical objects, counters, cubes etc to show how objects can be taken away.</p>  <p>$6 - 4 = 2$</p> <p>$4 - 2 = 2$</p>	 <p>$15 - 3 = 12$</p> <p>Cross out drawn objects to show what has been taken away.</p>	<p>$7 - 4 = 3$</p> <p>$16 - 9 = 7$</p>
Counting back	 <p>Move objects away from the group, counting backwards.</p> <p>Move the beads along the bead string as you count backwards.</p>	 <p>$5 - 3 = 2$</p> <p>Count back in ones using a number line.</p>	<p>Put 13 in your head, count back 4. What number are you at?</p>
Find the Difference	<p>Compare objects and amounts</p>  <p>'Seven is 3 more than four'</p> <p>'I am 2 years older than my sister'</p> <p>3 Pencils</p> <p>3 Erasers</p> <p>Lay objects to represent bar model.</p>	<p>Count on using a number line to find the difference.</p>  <p>+6</p>	<p>Hannah has 12 sweets and her sister has 5. How many more does Hannah have than her sister.?</p>

Y1 SUBTRACTION -

Objective & Strategy	Concrete	Pictorial	Abstract		
<p>Represent and use number bonds and related subtraction facts within 20</p> <p>Part Part Whole model</p>	 <p>Link to addition. Use PPW model to model the inverse.</p> <p>If 10 is the whole and 6 is one of the parts, what is the other part?</p> $10 - 6 = 4$	 <p>Use pictorial representations to show the part.</p>	<p>Move to using numbers within the part whole model.</p> 		
<p>Make 10</p>	<p>14-9</p>  <p>Make 14 on the ten frame. Take 4 away to make ten, then take one more away so that you have taken 5.</p>	<p>13-7</p>  <p>Jump back 3 first, then another 4. Use ten as the stopping point.</p>	<p>16-8</p> <p>How many do we take off first to get to 10? How many left to take off?</p>		
<p>Bar model</p>	 $5 - 2 = 3$		<table border="1" data-bbox="1375 1027 1809 1107"> <tr> <td>8</td> <td>2</td> </tr> </table> $10 = 8 + 2$ $10 = 2 + 8$ $10 - 2 = 8$ $10 - 8 = 2$	8	2
8	2				

Objective & Strategy	Concrete	Pictorial	Abstract
Regroup a ten into ten ones	 <p>Use a PV chart to show how to change a ten into ten ones, use the term 'take and make'</p>	 $20 - 4 = 16$	$20 - 4 = 16$
Partitioning to subtract without regrouping. <i>'Friendly numbers'</i>	$34 - 13 = 21$  <p>Use Dienes to show how to partition the number when subtracting without regrouping.</p>	Children draw representations of Dienes and cross off.  $43 - 21 = 22$	$43 - 21 = 22$
Make ten strategy <i>Progression should be crossing one ten, crossing more than one ten, crossing the hundreds.</i>	 $34 - 28$ <p>Use a bead bar or bead strings to model counting to next ten and the rest.</p>	 <p>Use a number line to count on to next ten and then the rest.</p>	$93 - 76 = 17$

Y2 SUBTRACTION -

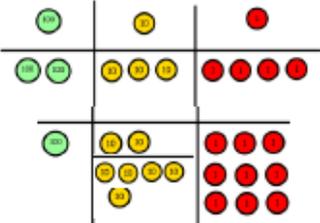
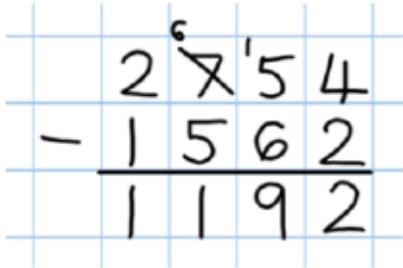
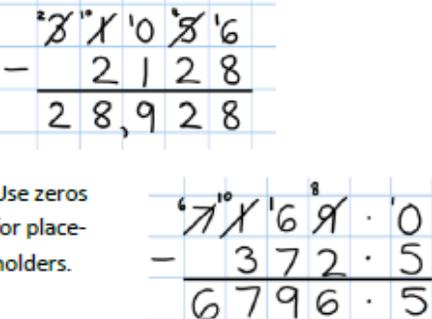
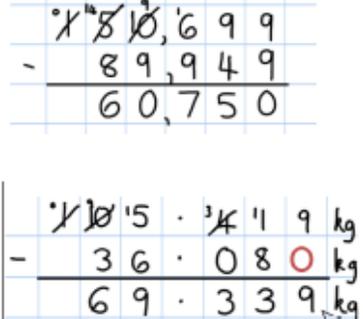
Objective & Strategy	Concrete	Pictorial	Abstract
Column subtraction without regrouping (friendly numbers)	 <p>47 - 32</p> <p>Use base 10 or Numicon to model</p>	 <p>Calculations</p> $\begin{array}{r} 47 \\ - 22 \\ \hline 25 \end{array}$ <p>Darw representations to support understanding</p>	$47 - 24 = 23$ $\begin{array}{r} 40 + 7 \\ - 20 + 4 \\ \hline 20 + 3 \end{array}$ <p>Intermediate step may be needed to lead to clear subtraction understanding.</p> 
Column subtraction with regrouping	 <p>Begin with base 10 or Numicon. Move to pv counters, modelling the exchange of a ten into ten ones. Use the phrase 'take and make' for exchange.</p>	$\begin{array}{r} 45 \\ - 29 \\ \hline 16 \end{array}$ <p>Tens Ones</p>  <p>Children may draw base ten or PV counters and cross off.</p>	$836 - 254 = 582$  <p>Begin by partitioning into pv columns</p> $728 - 582 = 146$  <p>Then move to formal method.</p>

Y3

SUBTRACTION -

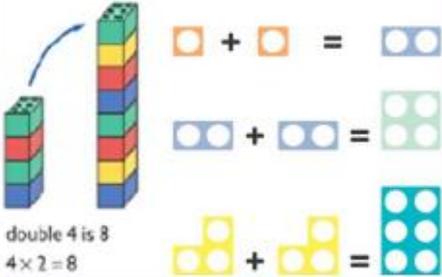
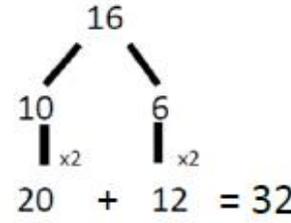
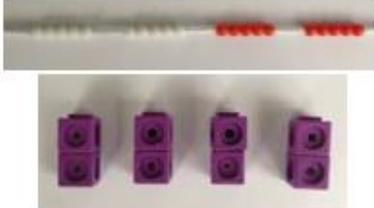
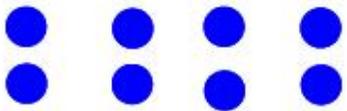
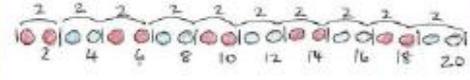
Y4-6

SUBTRACTION -

Objective & Strategy	Concrete	Pictorial	Abstract
Subtracting tens and ones Year 4 subtract with up to 4 digits. <i>Introduce decimal subtraction through context of money</i>	$234 - 179$  <p>Model process of exchange using Numicon, base ten and then move to PV counters.</p>	Children to draw pv counters and show their exchange—see Y3	 <p>Use the phrase 'take and make' for exchange</p>
Year 5- Subtract with at least 4 digits, including money and measures. <i>Subtract with decimal values, including mixtures of integers and decimals and aligning the decimal</i>	As Year 4	Children to draw pv counters and show their exchange—see Y3	 <p>Use zeros for place-holders.</p>
Year 6—Subtract with increasingly large and more complex numbers and decimal values.			

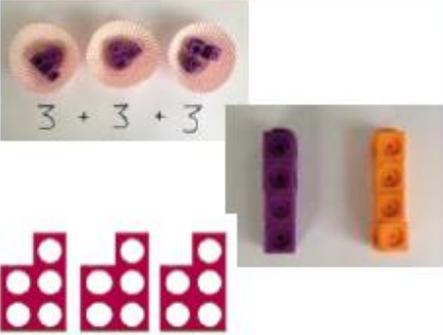
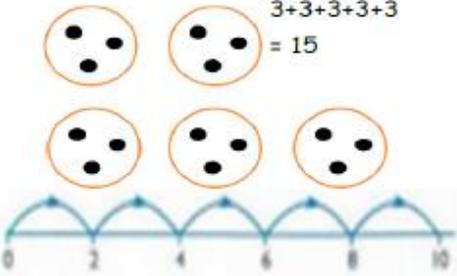
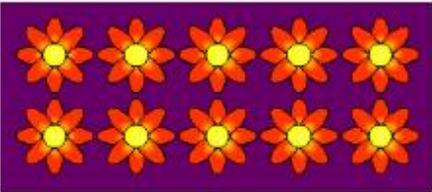
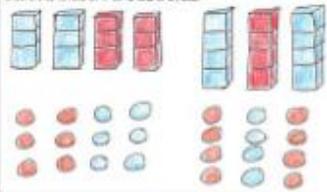
Y1

MULTIPLICATION X

Objective & Strategy	Concrete	Pictorial	Abstract
<p>Doubling</p>	<p>Use practical activities using manipulatives including cubes and Numicon to demonstrate doubling</p>  <p>double 4 is 8 $4 \times 2 = 8$</p>	<p>Draw pictures to show how to double numbers</p> <p>Double 4 is 8</p> 	<p>Partition a number and then double each part before recombining it back together.</p>  <p>$20 + 12 = 32$</p>
<p>Counting in multiples</p>	<p>Count the groups as children are skip counting, children may use their fingers as they are skip counting.</p> 	 <p>Children make representations to show counting in multiples.</p> 	<p>Count in multiples of a number aloud.</p> <p>Write sequences with multiples of numbers.</p> <p>2, 4, 6, 8, 10</p> <p>5, 10, 15, 20, 25, 30</p>
<p>Making equal groups and counting the total</p>	  <p>$\square \times \square = 8$</p> <p>Use manipulatives to create equal groups.</p>	<p>Draw  to show $2 \times 3 = 6$</p> <p>Draw and make representations</p>	<p>$2 \times 4 = 8$</p>

Y1

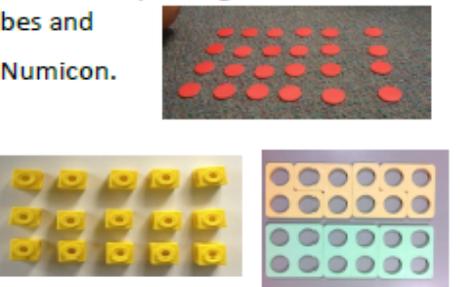
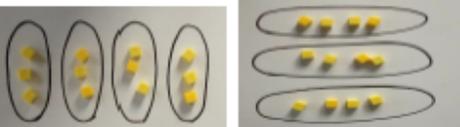
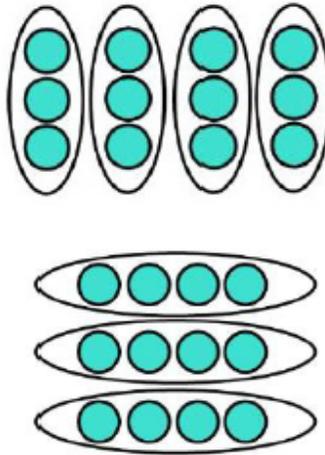
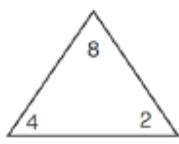
MULTIPLICATION X

Objective & Strategy	Concrete	Pictorial	Abstract
Repeated addition	 <p>Use different objects to add equal groups</p>	<p>Use pictorial including number lines to solve prob</p> <p>There are 3 sweets in one bag. How many sweets are in 5 bags altogether?</p> $3+3+3+3+3 = 15$ 	<p>Write addition sentences to describe objects and pictures.</p>  $2 + 2 + 2 + 2 + 2 = 10$
Understanding arrays	<p>Use objects laid out in arrays to find the answers to 2 lots 5, 3 lots of 2 etc.</p> 	<p>Draw representations of arrays to show understanding.</p> 	$3 \times 2 = 6$ $2 \times 5 = 10$

Y2 MULTIPLICATION X

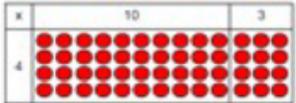
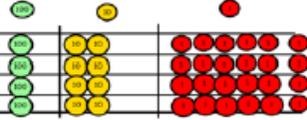
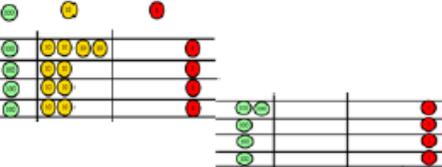
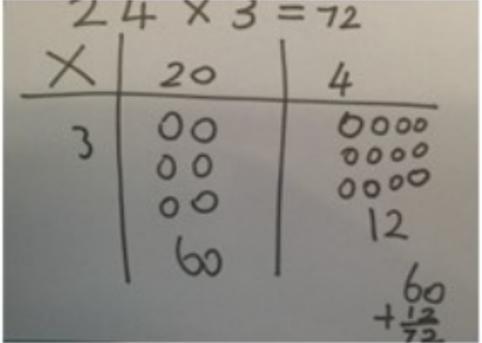
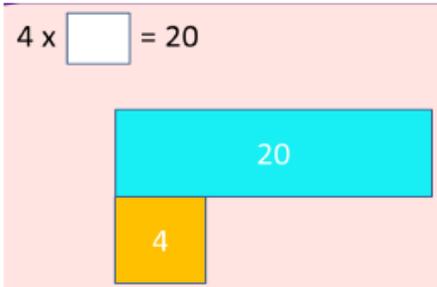
Y2

MULTIPLICATION X

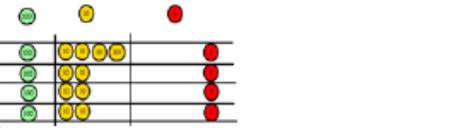
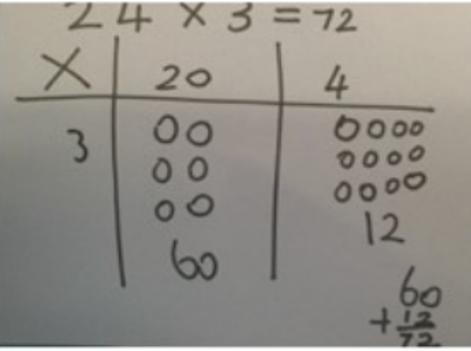
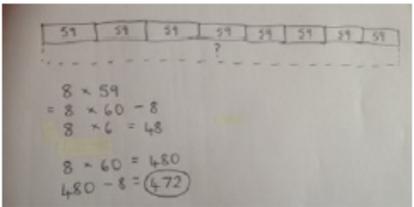
Objective & Strategy	Concrete	Pictorial	Abstract
<p>Multiplication is commutative</p>	<p>Create arrays using counters and cubes and Numicon.</p>  <p>Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.</p> 	<p>Use representations of arrays to show different calculations and explore commutativity.</p> 	$12 = 3 \times 4$ $12 = 4 \times 3$ <p>Use an array to write multiplication sentences and reinforce repeated addition.</p>  $5 + 5 + 5 = 15$ $3 + 3 + 3 + 3 + 3 = 15$ $5 \times 3 = 15$ $3 \times 5 = 15$
<p>Using the Inverse</p> <p><i>This should be taught alongside division, so pupils learn how they work alongside each other.</i></p>		 $\square \times \square = \square$ $\square \times \square = \square$ $\square \div \square = \square$ $\square \div \square = \square$	$2 \times 4 = 8$ $4 \times 2 = 8$ $8 \div 2 = 4$ $8 \div 4 = 2$ $8 = 2 \times 4$ $8 = 4 \times 2$ $2 = 8 \div 4$ $4 = 8 \div 2$ <p>Show all 8 related fact family sentences.</p>

Y3

MULTIPLICATION X

Objective & Strategy	Concrete	Pictorial	Abstract															
<p>Grid method</p>	<p>Show the links with arrays to first introduce the grid method.</p>  <p>4 rows of 10 4 rows of 3</p> <p>Move onto base ten to move towards a more compact method.</p>  <p>4 rows of 13</p> <p>Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows</p>  <p>Calculations 4×126</p> <p>Fill each row with 126</p>  <p>Calculations 4×126</p> <p>Add up each column, starting with the ones making any exchanges needed</p>  <p>Then you have your answer.</p>	<p>Children can represent their work with place value counters in a way that they understand.</p> <p>They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below.</p>  <p>Bar model are used to explore missing numbers</p> 	<p>Start with multiplying by one digit numbers and showing the clear addition alongside the grid.</p> <table border="1" data-bbox="1467 391 1780 478"> <tr> <td>X</td> <td>30</td> <td>5</td> </tr> <tr> <td>7</td> <td>210</td> <td>35</td> </tr> </table> <p>$210 + 35 = 245$</p> <p>Moving forward, multiply by a 2 digit number showing the different rows within the grid method.</p> <table border="1" data-bbox="1467 750 1769 933"> <tr> <td></td> <td>10</td> <td>8</td> </tr> <tr> <td>10</td> <td>100</td> <td>80</td> </tr> <tr> <td>3</td> <td>30</td> <td>24</td> </tr> </table>	X	30	5	7	210	35		10	8	10	100	80	3	30	24
X	30	5																
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	10	8																
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3	30	24																

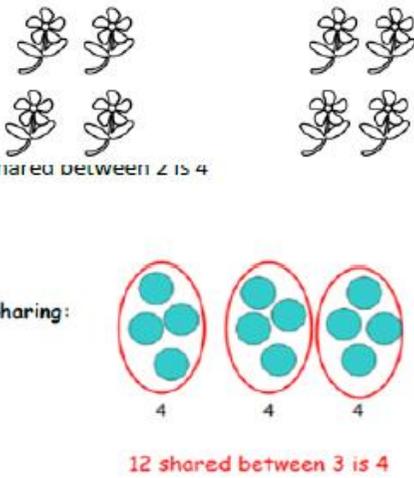
Y4 MULTIPLICATION X

Objective & Strategy	Concrete	Pictorial	Abstract																																				
<p>Grid method recap from year 3 for 2 digits x 1 digit</p> <p>Move to multiplying 3 digit numbers by 1 digit. (year 4 expectation)</p>	<p>Use place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows</p>  <p>Calculations 4×126</p> <p>Fill each row with 126</p>  <p>Add up each col. making any exchanges needed</p>	<p>Children can represent their work with place value counters in a way that they understand.</p> <p>They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below.</p> 	<p>Start with multiplying by one digit numbers and showing the clear addition alongside the grid.</p> <table border="1" data-bbox="1478 391 1780 478"> <tr> <td>x</td> <td>30</td> <td>5</td> </tr> <tr> <td>7</td> <td>210</td> <td>35</td> </tr> </table> <p>$210 + 35 = 245$</p>	x	30	5	7	210	35																														
x	30	5																																					
7	210	35																																					
<p>Column multiplication</p>	<p>Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. $321 \times 2 = 642$</p> <table border="1" data-bbox="383 965 705 1332"> <thead> <tr> <th>Hundreds</th> <th>Tens</th> <th>Ones</th> </tr> </thead> <tbody> <tr> <td>3</td> <td>2</td> <td>1</td> </tr> <tr> <td>3</td> <td>2</td> <td>1</td> </tr> </tbody> </table> <p>It is important at this stage that they always multiply the ones first.</p> <p>The corresponding long multiplication is modelled alongside</p>	Hundreds	Tens	Ones	3	2	1	3	2	1	<table border="1" data-bbox="963 861 1299 941"> <tr> <td>x</td> <td>300</td> <td>20</td> <td>7</td> </tr> <tr> <td>4</td> <td>1200</td> <td>80</td> <td>28</td> </tr> </table> <p>The grid method may be used to show how this relates to a formal written method.</p>  <p>Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.</p>	x	300	20	7	4	1200	80	28	<table data-bbox="1534 837 1713 1189"> <tr> <td>327</td> </tr> <tr> <td>x 4</td> </tr> <tr> <td>1308</td> </tr> </table> <p>This may lead to a compact method.</p> <table border="1" data-bbox="1422 1220 1646 1388"> <tr> <td></td> <td>3</td> <td>2</td> <td>7</td> </tr> <tr> <td>x</td> <td></td> <td></td> <td>4</td> </tr> <tr> <td></td> <td>1</td> <td>3</td> <td>0</td> </tr> <tr> <td></td> <td></td> <td>1</td> <td>2</td> </tr> </table>	327	x 4	1308		3	2	7	x			4		1	3	0			1	2
Hundreds	Tens	Ones																																					
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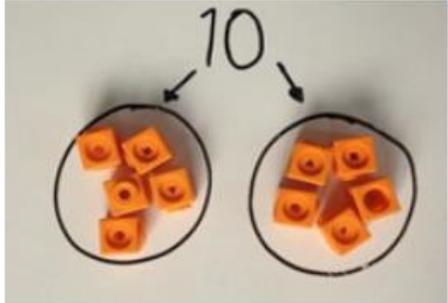
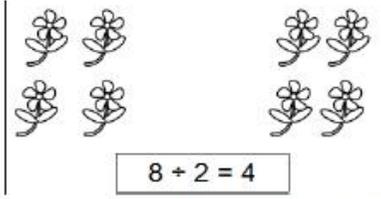
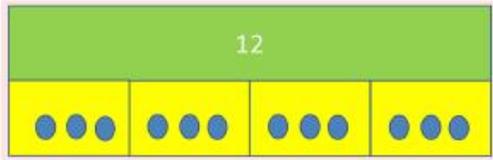
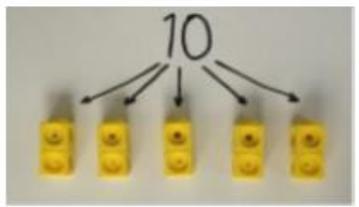
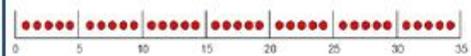
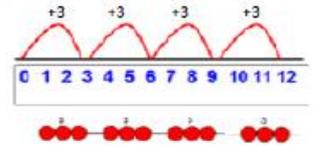
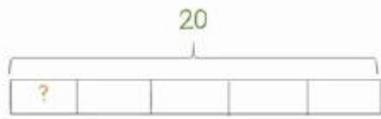
Y6

MULTIPLICATION X

Objective & Strategy	Concrete	Pictorial	Abstract
Multiplying decimals up to 2 decimal places by a single digit.			<p>Remind children that the single digit belongs in the units column. Line up the decimal points in the question and the answer.</p> $\begin{array}{r} 3.19 \\ \times 8 \\ \hline 25.52 \end{array}$

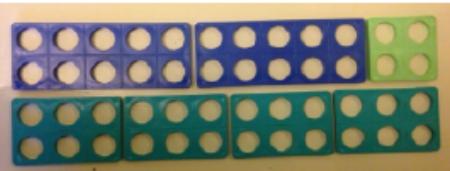
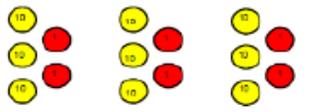
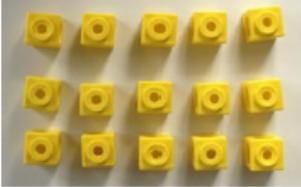
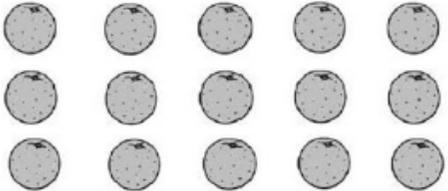
Objective & Strategy	Concrete	Pictorial	Abstract
<p>Division as sharing</p> <p>Use Gordon ITPs for modelling</p>	 <p>I have 10 cubes, can you share them equally in 2 groups?</p>	<p>Children use pictures or shapes to share quantities.</p>  <p>8 shared between 2 is 4</p> <p>Sharing:</p> <p>12 shared between 3 is 4</p>	<p>12 shared between 3 is</p> <p>4</p>

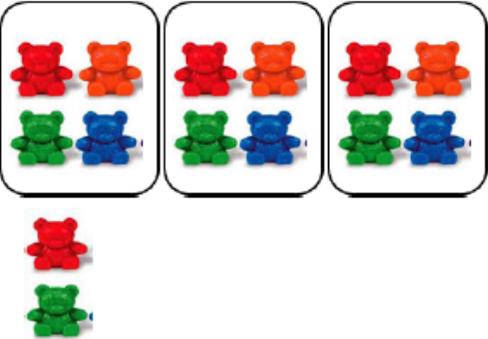
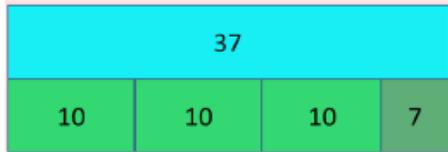
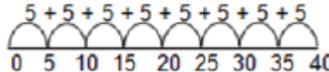
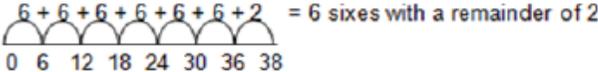
Y2 DIVISION ÷

Objective & Strategy	Concrete	Pictorial	Abstract
<p>Division as sharing</p>	 <p>I have 10 cubes, can you share them equally in 2 groups?</p>	<p>Children use pictures or shapes to share quantities.</p>  <p>$8 \div 2 = 4$</p> <p>Children use bar modelling to show and support understanding.</p>  <p>$12 \div 4 = 3$</p>	<p>$12 \div 3 = 4$</p>
<p>Division as grouping</p>	<p>Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.</p>  	<p>Use number lines for grouping</p>  <p>$12 \div 3 = 4$</p> <p>Think of the bar as a whole, split it into the number of groups you are dividing by and work out how many would be within each group.</p>  <p>$20 \div 5 = ?$ $5 \times ? = 20$</p>	<p>$28 \div 7 = 4$</p> <p>Divide 28 into 7 groups. How many are in each group?</p>

Y3

DIVISION ÷

Objective & Strategy	Concrete	Pictorial	Abstract
Division as grouping	<p>Use cubes, counters, objects or place value counters to aid understanding.</p>  <p>24 divided into groups of 6 = 4</p> $96 \div 3 = 32$ 	<p>Continue to use bar modelling to aid solving division problems.</p>  <p>$20 \div 5 = ?$ $5 \times ? = 20$</p>	<p>How many groups of 6 in 24?</p> $24 \div 6 = 4$
Division with arrays	 <p>Link division to multiplication by creating an array and thinking about the number sentences that can be created.</p> <p>Eg $15 \div 3 = 5$ $5 \times 3 = 15$ $15 \div 5 = 3$ $3 \times 5 = 15$</p>	<p>Draw an array and use lines to split the array into groups to make multiplication and division sentences</p> 	<p>Find the inverse of multiplication and division sentences by creating eight linking number sentences.</p> $7 \times 4 = 28$ $4 \times 7 = 28$ $28 \div 7 = 4$ $28 \div 4 = 7$ $28 = 7 \times 4$ $28 = 4 \times 7$ $4 = 28 \div 7$ $7 = 28 \div 4$

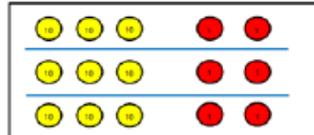
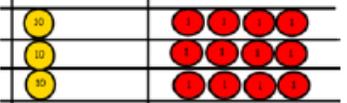
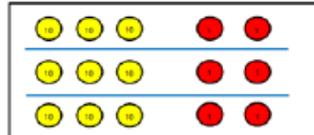
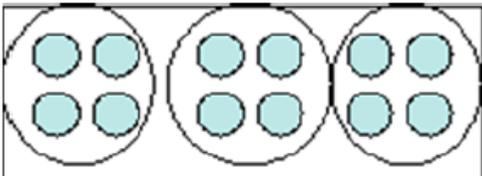
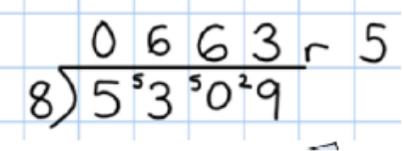
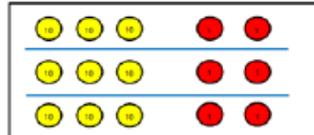
Objective & Strategy	Concrete	Pictorial	Abstract
Division with remainders.	<p>$14 \div 3 =$</p> <p>Divide objects between groups and see how much is left over</p> 	<p>Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.</p>  <p>Draw dots and group them to divide an amount and clearly show a remainder.</p>  <p>Use bar models to show division with remainders.</p>  <p>Example without remainder: $40 \div 5$ Ask "How many 5s in 40?" $5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 8 \text{ fives}$</p>  <p>Example with remainder: $38 \div 6$</p>  <p>For larger numbers, when it becomes inefficient to count in single multiples, bigger jumps can be recorded using known facts.</p>	<p>Complete written divisions and show the remainder using r.</p> $\begin{array}{ccccccc} 29 \div 8 = 3 \text{ REMAINDER } 5 \\ \uparrow \quad \uparrow \quad \uparrow \quad \quad \quad \uparrow \\ \text{dividend} \quad \text{divisor} \quad \text{quotient} \quad \quad \quad \text{remainder} \end{array}$

Y3

DIVISION ÷

Y4-6

DIVISION ÷

Objective & Strategy	Concrete	Pictorial	Abstract									
<p>Divide at least 3 digit numbers by 1 digit.</p> <p>Short Division</p>	<p>$96 \div 3$</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td></td> <td style="text-align: center;">Tens</td> <td style="text-align: center;">Units</td> </tr> <tr> <td></td> <td style="text-align: center;">3</td> <td style="text-align: center;">2</td> </tr> <tr> <td style="text-align: center;">3</td> <td style="text-align: center;">  </td> <td></td> </tr> </table> <p>Use place value counters to divide using the bus stop method alongside</p> <div style="display: flex; align-items: center; margin-left: 20px;">  <div style="margin-left: 20px;"> <p>Calculations</p> $42 \div 3$ </div> </div> <div style="margin-left: 20px;">  </div> <p>$42 \div 3 =$</p> <p>Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.</p> <div style="display: flex; align-items: center; margin-left: 20px;">  </div> <div style="margin-left: 20px;">  </div> <p>We exchange this ten for ten ones and then share the ones equally among the groups.</p> <div style="display: flex; align-items: center; margin-left: 20px;">  </div> <p>We look how much in 1 group so the answer is 14.</p>		Tens	Units		3	2	3			<p>Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.</p> <div style="text-align: center;">  </div> <p>Encourage them to move towards counting in multiples to divide more efficiently.</p>	<p>Begin with divisions that divide equally with no remainder.</p> $\begin{array}{r} 218 \\ 3 \overline{) 872} \end{array}$ <p>4</p> <p>Move onto divisions with a remainder.</p> $\begin{array}{r} 86 \text{ r } 2 \\ 3 \overline{) 432} \end{array}$ <p>5</p> <p>Finally move into decimal places to divide the total accurately.</p> $\begin{array}{r} 14.6 \\ 35 \overline{) 511.0} \end{array}$ <div style="margin-top: 20px;">  </div>
	Tens	Units										
	3	2										
3												

Long Division

Step 1—a remainder in the ones

$$\begin{array}{r} \text{h t o} \\ 041 \text{ R}1 \\ \hline 4 \overline{) 165} \end{array}$$

4 does not go into 1 (hundred). So combine the 1 hundred with the 6 tens (160).

4 goes into 16 four times.

4 goes into 5 once, leaving a remainder of 1.

$$\begin{array}{r} \text{th h t o} \\ 0400 \text{ R}7 \\ \hline 8 \overline{) 3207} \end{array}$$

8 does not go into 3 of the thousands. So combine the 3 thousands with the 2 hundreds (3,200).

8 goes into 32 four times ($3,200 \div 8 = 400$)

8 goes into 0 zero times (tens).

8 goes into 7 zero times, and leaves a remainder of 7.

Long Division

Step 1 continued...

$$\begin{array}{r} \text{h t o} \\ 061 \\ 4 \overline{) 247} \\ \underline{-4} \\ 3 \end{array}$$

When dividing the ones, 4 goes into 7 one time. Multiply $1 \times 4 = 4$, write that four under the 7, and subtract. This finds us the remainder of 3.

Check: $4 \times 61 + 3 = 247$

$$\begin{array}{r} \text{th h t o} \\ 0402 \\ 4 \overline{) 1609} \\ \underline{-8} \\ 1 \end{array}$$

When dividing the ones, 4 goes into 9 two times. Multiply $2 \times 4 = 8$, write that eight under the 9, and subtract. This finds us the remainder of 1.

Check: $4 \times 402 + 1 = 1,609$

Long Division

Step 2—a remainder in the tens

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
$\begin{array}{r} \text{t o} \\ 2 \overline{)58} \end{array}$	$\begin{array}{r} \text{t o} \\ 2 \overline{)58} \\ -4 \\ \hline 1 \end{array}$	$\begin{array}{r} \text{t o} \\ 2 \overline{)58} \\ -4 \downarrow \\ \hline 18 \end{array}$
<p>Two goes into 5 two times, or 5 tens $\div 2 = 2$ whole tens -- but there is a remainder!</p>	<p>To find it, multiply $2 \times 2 = 4$, write that 4 under the five, and subtract to find the remainder of 1 ten.</p>	<p>Next, drop down the 8 of the ones next to the leftover 1 ten. You combine the remainder ten with 8 ones, and get 18.</p>

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
$\begin{array}{r} \text{t o} \\ 2 \overline{)58} \\ -4 \\ \hline 18 \end{array}$	$\begin{array}{r} \text{t o} \\ 2 \overline{)58} \\ -4 \\ \hline 18 \\ -18 \\ \hline 0 \end{array}$	$\begin{array}{r} \text{t o} \\ 2 \overline{)58} \\ -4 \\ \hline 18 \\ -18 \\ \hline 0 \end{array}$
<p>Divide 2 into 18. Place 9 into the quotient.</p>	<p>Multiply $9 \times 2 = 18$, write that 18 under the 18, and subtract.</p>	<p>The division is over since there are no more digits in the dividend. The quotient is 29.</p>

Y6

DIVISION \div

Y6

DIVISION ÷

Long Division

Step 2—a remainder in any of the place values

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
$\begin{array}{r} \text{h t o} \\ 1 \\ 2 \overline{) 278} \\ -2 \\ \hline 07 \end{array}$ <p>Two goes into 2 one time, or 2 hundreds ÷ 2 = 1 hundred.</p>	$\begin{array}{r} \text{h t o} \\ 1 \\ 2 \overline{) 278} \\ -2 \\ \hline 0 \end{array}$ <p>Multiply $1 \times 2 = 2$, write that 2 under the two, and subtract to find the remainder of zero.</p>	$\begin{array}{r} \text{h t o} \\ 18 \\ 2 \overline{) 278} \\ -2 \\ \hline 07 \end{array}$ <p>Next, drop down the 7 of the tens next to the zero.</p>
Divide.	Multiply & subtract.	Drop down the next digit.
$\begin{array}{r} \text{h t o} \\ 13 \\ 2 \overline{) 278} \\ -2 \\ \hline 07 \end{array}$ <p>Divide 2 into 7. Place 3 into the quotient.</p>	$\begin{array}{r} \text{h t o} \\ 13 \\ 2 \overline{) 278} \\ -2 \\ \hline 07 \\ -6 \\ \hline 1 \end{array}$ <p>Multiply $3 \times 2 = 6$, write that 6 under the 7, and subtract to find the remainder of 1 ten.</p>	$\begin{array}{r} \text{h t o} \\ 13 \\ 2 \overline{) 278} \\ -2 \\ \hline 07 \\ -6 \\ \hline 18 \end{array}$ <p>Next, drop down the 8 of the ones next to the 1 leftover ten.</p>
1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
$\begin{array}{r} \text{h t o} \\ 139 \\ 2 \overline{) 278} \\ -2 \\ \hline 07 \\ -6 \\ \hline 18 \end{array}$ <p>Divide 2 into 18. Place 9 into the quotient.</p>	$\begin{array}{r} \text{h t o} \\ 139 \\ 2 \overline{) 278} \\ -2 \\ \hline 07 \\ -6 \\ \hline 18 \\ -18 \\ \hline 0 \end{array}$ <p>Multiply $9 \times 2 = 18$, write that 18 under the 18, and subtract to find the remainder of zero.</p>	$\begin{array}{r} \text{h t o} \\ 139 \\ 2 \overline{) 278} \\ -2 \\ \hline 07 \\ -6 \\ \hline 18 \\ -18 \\ \hline 0 \end{array}$ <p>There are no more digits to drop down. The quotient is 139.</p>